THEORETICAL STUDY OF ELECTRON CONCENTRATION DISTRIBUTION IN POSITIVE COLUMN GLOW DISCHARGE WITH LONGITUDINAL GAS STREAM

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A solution is obtained to the system of equations describing the positive column of a glow discharge in a cylindrical channel with a longitudinal gas stream. Expressions are derived for calculating the electron concentration and the electric field intensity.

A comprehensive study of plasma generation in a glow discharge in gas streams is very timely now, in connection with efforts made to produce gaseous lasers with convective cooling and plasmochemical reactors. Much attention is, therefore, paid to building discharge chambers with glow discharge and to studying their characteristics. Existing discharge chambers are characterized by a diversity of shapes and sizes as well as of discharge and stream parameters. This is attributable to both the novelty of the problem and to practical requirements, but also to the lack of a general method of analysis of physical processes in a glow discharge [1]. Such a method would facilitate the design of optimum discharge chambers and the calculation of processes occurring in them. Accordingly, one of the main things a theory of glow discharge must do is establish the dependence of the electron concentration, electron temperature, electric field intensity, and several other internal characteristics on the pressure, velocity, temperature, and degree of turbulence of the stream as well as on the dimensions of the discharge chamber, the current, and the degree of preionization. At the present time, however, there is no theory available which takes all these factors into account and could serve as the basis for design calculations pertaining to discharge chambers and be sufficiently accurate for practical purposes [2].

Some properties of discharges in a transverse stream are describable by expressions obtained in [3] from the solution of the one-dimensional equation of continuity for the electron gas. It has been demonstrated in [3], specifically, that during passage of a gas stream through a discharge chamber, the discharge shifts downstream and the electric field intensity increases with the electron concentration distribution ceasing to be symmetric with respect to the plane of the electrodes. A discharge of cylindrical form in a longitudinal stream was considered in [4]. The authors of that study assumed the properties of the discharge to be uniform along the cylinder axis and the fluctuational motion of ions to be completely correlated with that of neutral particles. They solved the equation of discontinuity for the electron gas, taking into account the dependence of the coefficient of ambipolar diffusion on the degree of turbulence of the stream. An analysis of their solution reveals that as the Reynolds number increases, the ionization rate and the electric field intensity increase while the radial profile of electron concentration becomes fuller. Under real conditions the properties of a gas discharge also vary appreciably in the direction of the gas flow. In known theoretical studies of the plasma in glow discharge of cylindrical geometry, hardly any consideration has been given to the manner of buildup of the discharge following its interaction with the gas stream. The object of this study will, therefore, be to continue with the theory of glow discharge in a cylindrical channel with a gas stream.

With the aid of the expressions for electron stream and ion stream intensities

$$\mathbf{\Gamma}_e = n_e\mathbf{V} - D_e\nabla n_e - n_e\mu_e\mathbf{E},$$
$$\mathbf{\Gamma}_i = n_i\mathbf{V} - D_i\nabla n_i + n_i\mu_i\mathbf{E},$$

and the condition of quasineutrality, also assuming a constant velocity of the gas in the discharge channel, one can reduce the steady-state equation of continuity for electrons in a plasma not containing negative ions to the form

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$$\frac{\partial \bar{n}_e}{\partial \bar{z}} = \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{n}_e}{\partial \bar{r}} \right) + \beta(\bar{z}) \bar{n}_e - \frac{n_e(0,\ 0) R^2 \delta}{D_a} \bar{n}_e^2. \tag{1}$$

Here

$$\bar{r} = \frac{r}{R};\ \bar{z} = \frac{1}{\mathrm{Pe}_d} \frac{z}{R};\ \beta(\bar{z}) = \frac{\nu(\bar{z}) R^2}{D_a};$$

$$\bar{n}_e(\bar{r},\ \bar{z}) = \frac{n_e(\bar{r},\ \bar{z})}{n_e(0,\ 0)};\ \mathrm{Pe}_d = \frac{VR}{D_a};\ T = \frac{n_e(0,\ 0) R^2 \delta}{D_a}$$

The dimensionless number T characterizes the ratio of loss of electrons due to volume recombination to loss of electrons due to ambipolar diffusion. When $D_\alpha/R^2 >> n_e(0,\ 0)\delta$, then the effect of recombination processes to the electron balance can be disregarded and Eq. (1) can be rewritten as

$$\frac{\partial \bar{n}_e}{\partial \bar{z}} = \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{n}_e}{\partial \bar{r}} \right) + \beta(\bar{z}) \bar{n}_e. \tag{1a}$$

The magnitudes of current, electric field intensity, and electron concentration are related through Ohm's law in integral form:

$$I = 2\pi e \mu_e E n_e(0,\ 0) R^2 \int_0^1 \bar{n}_e \bar{r} d\bar{r}. \tag{2}$$

The ionization frequency ν is a function of E/p [5]. Considering that the variation of pressure p along the discharge channel under conditions of glow discharge is small [6], we regard ν as a function of E only:

$$\nu = f(E). \tag{3}$$

The pressure will then appear in the solution as a parameter. The system of Eqs. (1a)-(3) will be solved for the conditions

$$\bar{n}_e(\bar{r},\ 0) = \varphi(\bar{r}),\ \bar{n}_e(\bar{z},\ 1) = f(\bar{z}),\ \frac{\partial \bar{n}_e}{\partial \bar{r}}(\bar{z},\ 0) = 0, \tag{4}$$

where $0 \leqslant \bar{r} \leqslant 1$ and $\bar{z} \geqslant 0$. Taking (4) into account, the electron concentration at the channel wall generally is not zero, but it can be very low [5].

Equation (1a) will be solved with the aid of the Hankel finite integral transformation

$$N(\lambda_n,\ \bar{z}) = \int_0^1 \bar{r} J_0(\lambda_n \bar{r}) \bar{n}_e(\bar{r},\ \bar{z}) d\bar{r}. \tag{5}$$

Here J_0 is the zeroth-order Bessel function and λ_n are roots of the equation $J_0(\lambda) = 0$. The inverse transformation from transform N to the original $\bar{n}_e$ is performed according to the relation

$$\bar{n}_e(\bar{r},\ \bar{z}) = \sum_{n=1}^{\infty} \frac{2J_0(\lambda_n \bar{r})}{J_1^2(\lambda_n)} N(\lambda_n \bar{z}). \tag{6}$$

Applying the transformation (5) to Eq. (1a) and taking into account conditions (4), we find

$$\frac{dN}{d\bar{z}} + [\lambda_n^2 - \beta(\bar{z})] N = \lambda_n J_1(\lambda_n) f(\bar{z}). \tag{7}$$

The well-known solution to this equation for the initial condition

$$N(\lambda_n,\ 0) = \int_0^1 \bar{r} J_0(\lambda_n,\ \bar{r}) \varphi(\bar{r})\, d\bar{r}$$

can be written as

$$N(\lambda_n,\ \bar{z}) = \left[\int_0^1 \bar{r} J_0(\lambda_n,\ \bar{r}) \varphi(\bar{r}) d\bar{r} + \lambda_n J_1(\lambda_n) \int_0^{\bar{z}} f(\theta) B_n(\theta)\, d\theta \right] B_n^{-1}(\bar{z}), \tag{8}$$

where

$$B_n(\theta) = \exp\left[\lambda_n^2\theta - \int_0^\theta \beta(t)\,dt\right].$$

Inserting solution (8) into expression (6) yields the solution to Eq. (1a) for any laws of β and E variation. In order to solve the system (1a)-(3), it is necessary to determine E and β as functions of $\bar{z}$ under the conditions of the problem. We will do it for f(z) = 0. For obtaining the final expression we will use the well-known dependence of ν on E in the form $\nu = cE^2$ [6], where c is a constant. Then $\beta(\bar{z}) = bE^2(\bar{z})$ with $b = cR^2/D_a$ and expressions (2), (6), (8) yield

$$\bar{n}_e(\bar{r},\ \bar{z}) = \exp\left(b\int_0^{\bar{z}} E^2 d\bar{z}\right)\sum_{n=1}^{\infty} A_n J_0(\lambda_n \bar{r})\exp(-\lambda_n^2 \bar{z}), \tag{9}$$

$$I = \pi e\mu_e n_e(0,\ 0)\, R^2 E(\bar{z})\, Q(\bar{z}) \exp\left(b\int_0^{\bar{z}} E^2 d\bar{z}\right), \tag{10}$$

where

$$A_n = \frac{2}{J_1^2(\lambda_n)}\int_0^1 \bar{r} J_0(\lambda_n \bar{r})\,\varphi(\bar{r})\,d\bar{r};$$

$$Q(\bar{z}) = 2\sum_{n=1}^{\infty}\frac{A_n J_1(\lambda_n)}{\lambda_n}\exp(-\lambda_n^2 \bar{z}).$$

The solution to the nonlinear equation (10) for the electric field intensity is [7]

$$E(\bar{z}) = I\left[Q^2\left(2I^2 b\int_0^{\bar{z}} Q^{-2} d\bar{z} + \pi^2 e^2 \mu_e^2 n_e^2(0, 0) R^4\right)\right]^{-0,5}. \tag{11}$$

Inserting solution (11) into expression (9) yields for the electron concentration distribution

$$\bar{n}_e(\bar{r},\ \bar{z}) = \frac{I\sum_{n=1}^{\infty} A_n J_0(\lambda_n \bar{r})\exp(-\lambda_n^2 \bar{z})}{\pi e\mu_e R^2 n_e(0,\ 0)\, E(\bar{z})\, Q(\bar{z})}. \tag{12}$$

It follows from (11) that, as $\bar{z}$ increases, the electric field intensity tends to a limit. The expression for $E(\bar{z})$, moreover, represents an indeterminacy of the ∞/∞ kind

$$E_\infty^2 = \lim_{\bar{z}\to\infty}\frac{I^2}{Q^2(\bar{z})\left[2I^2 b\int_0^{\bar{z}}\frac{d\bar{z}}{Q^2(z)} + \pi^2 e^2\mu_e^2 n_e^2(0,\ \ 0) R^4\right]}.$$

Application of l'Hospital's rule, with the expression for $Q(\bar{z})$ taken into account, yields

$$E_\infty = \frac{\lambda_1}{R}\left(\frac{D_a}{c}\right)^{0,5}. \tag{13}$$

For the ultimate electron concentration distribution expressions (9), (12), and (13) yield

$$n_{e\infty}(\bar{r})\ \frac{IJ_0(\lambda_1 \bar{r})}{2\pi e\mu_e R J_1(\lambda_1)}\left(\frac{c}{D_a}\right)^{0,5}. \tag{14}$$

On the basis of the adopted dependence of ν on E, we have thus obtained expressions for calculating the electric field intensity (11) and the electron concentration (12) also for the ultimate electric field intensity E_∞ (13) and electron concentration $n_\infty(\bar{r})$ (14). In the special case of $\varphi(\bar{r}) = J_0(\lambda_1 \bar{r})$ the expressions for the electric field intensity and the mean-over-cross-section electron concentration become

$$E(\bar{z}) = E_\infty\left[1 + \left(\frac{E_\infty^2}{E_0^2} - 1\right)\exp(-2\lambda_1^2 \bar{z})\right]^{-0,5}, \tag{15}$$

$$\bar{n}_{em}(\bar{z}) = \frac{I}{\pi e\mu_e R^2 n_e(0,\ 0)\, E(\bar{z})}. \tag{16}$$

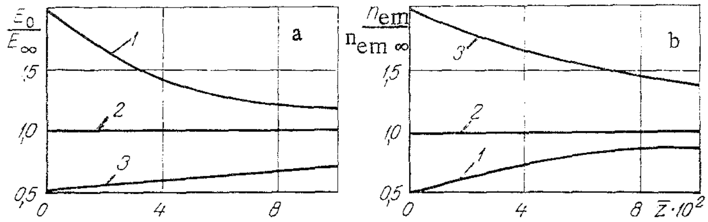

Fig. 1. Variation of the dimensionless electric field intensity E_0/E_∞ (a) and of the mean-over-cross-section electron concentration $n_{em}/n_{em\ \infty}$ (b) along the $\bar{z}$ axis; E_0/E_∞: 1) 2; 2) 1; 3) 0.5.

With the aid of these expressions we will now examine the laws governing the buildup of glow discharge in the direction of gas flow. Depending on the relation between the magnitudes of E_0 and E_∞, the following cases can occur.

1. $E_0 > E_\infty$. According to expression (15), E decreases with increasing $\bar{z}$ and tends to E_∞ while $\bar{n}_{em}$ increases and tends to the limit $n_{em\ \infty}$ (curves 1 in Fig. 1). This condition is found in the case of weak preionization of the gas entering the discharge channel.

2. $E_0 = E_\infty$. In this case neither E nor $\bar{n}_{em}$ varies along the $\bar{z}$ axis (curves 2 in Fig. 1).

3. $E_0 < E_\infty$. According to expressions (15) and (16), E increases and $\bar{n}_{em}$ decreases along the $\bar{z}$ axis (curves 3 in Fig. 1). This condition is found in the case of strong preionization of the gas entering the discharge channel.

According to expressions (15) and (16), the rate at which E and $\bar{n}_{em}$ tend to their respective limits increases with decreasing Peclet number Pe_d, i.e., with decreasing velocity of the gas and increasing coefficient of ambipolar diffusion.

It thus follows from the solution obtained in this study that in a glow discharge in a longitudinal gas stream, the electron concentration and the electric field intensity generally vary along the channel. There exists a limiting segment where the electric field intensity remains the same and the electron concentration distribution over the column section is describable by a Bessel function. The rate of discharge buildup in the stream increases with decreasing velocity of the gas and increasing coefficient of ambipolar diffusion. In the special case, Schottky's results [8] are obtained for a nonflow glow discharge.

NOTATION

r, z, respectively, the radial coordinate and the longitudinal coordinate; R, channel radius; I, current; E, electric field intensity; V, gas velocity; Pe_d, Peclet diffusion number; ν, ionization frequency; n, particle concentration; n_m, mean-over-cross-section particle concentration; D, diffusion coefficient; D_α, coefficient of ambipolar diffusion; e, charge of an electron; μ, mobility; p, pressure; δ, recombination factor; subscripts: i and e refer, respectively, to ions and electrons; 0 and ∞, to values at $\bar{z} = 0$ and $\bar{z} = \infty$, respectively; the dash above a symbol refers to a dimensionless quantity.

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